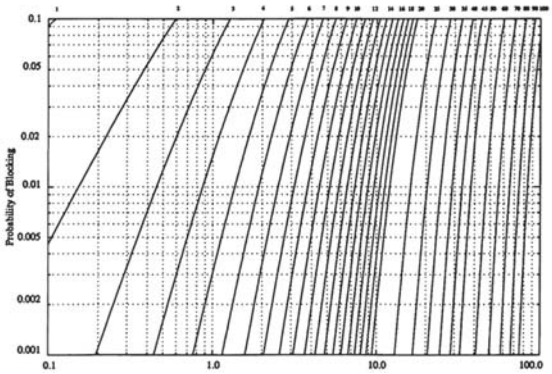
ECS455 Chapter 2

Cellular Systems

2.4 Traffic Handling Capacity and Erlang B Formula

Dr.Prapun Suksompong
prapun.com/ecs455



Capacity Concept: A Revisit

- Q: If I have *m* channels per cell, is it true that my cell can support only *m* users?
- A: Yes and No
- Let's try one example.
- How often do you make a call?
  - 3 calls a day, on average. ← $\lambda_u = 3 \frac{\text{calls}}{\text{day}}$
- How long is the call?
  - 10 mins (per call), on average. ← $H = \frac{1}{\mu} = 10 \frac{\text{mins}}{\text{call}}$
- So, one person uses

$= \lambda_u \times H = \lambda_u / \mu$

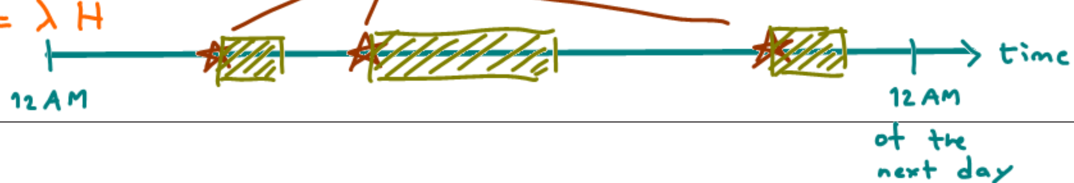
$A_u = 3 \frac{\cancel{\text{calls}}}{\text{day}} \times \frac{10 \text{ mins}}{\cancel{\text{call}}} = \frac{\cancel{30}\ \text{mins}}{24 \times \cancel{60}_{2}\ \text{mins}} = \frac{1}{48} \approx 0.021$ [Erlang]

(2%)

$n\times \downarrow \quad \downarrow \times n$

$A = \lambda H$

call initiation times



Capacity Concept: A Revisit

- If we can "give" the time that "User 1" is idle to other users,
  - then one channel can support $\approx \frac{100\%}{2\%} = 50$ users!! $\left(\frac{1}{1/48} = 48\right)$

50x increase in capacity?

- True?

Capacity Concept: A Revisit

- If we can "give" the time that "User 1" is idle to other users,
  - then one channel can support users!!
- True? Not quite.
  - 50 users are OK if we have a way to "manipulate" all of them to not make calls when another user is using the channel.
  - Real users access the channel randomly.
    - (Call initialization/request times are random.)
  - So, can guarantee service for only one user.
  - If we allow > 1 users, then we (and the users) will have to deal with congestion.

busy signal
blocked call

New Concepts for a New Look at Capacity

- We can let more than one user share a channel by using it at different times.
- **Blocked call** happens if a user requests to make a call when all the channels are occupied by other users.
- **Probability of (call) blocking: P_b**
  - The likelihood that a call is blocked because there is no available channel.
  - 1%, 2%, 5%
- In which case, the number of users that a cell can support can exceed $S/_N$.
  - How much larger depends strongly on the value of P_b that can be tolerated.

Trunking

- Allow a large number (n) of users to **share** the relatively small number (m) of channels in a cell (or a sector) by providing access to each user, **on demand**, from a **pool** of available channels.

	Omnidirectional	120° Sectoring	60° Sectoring
#sectors/cell	1	3	6
#channels/sector	$m = \lfloor S/N \rfloor$	$m = \left\lfloor \frac{S}{N}/3 \right\rfloor$	$m = \left\lfloor \frac{S}{N}/6 \right\rfloor$

- Exploit the **statistical behavior** of users.
- Each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.

Common Terms (1)

- **Traffic Intensity**: Measure of channel time utilization (traffic load / amount of traffic), which is the average channel occupancy measured in **Erlangs**.
  - Dimensionless
  - Denoted by A.
- **Holding Time**: Average duration of a typical call.
  - Denoted by $H = 1/\mu$.
- **Request Rate**: The average number of call requests per unit time. Denoted by λ.
- Use A_u and λ_u to denote the corresponding quantities for one user.
- Note that $A = nA_u$ and $\lambda = n\lambda_u$ where n is the number of users supported by the pool (trunked channels) under consideration.

Common Terms (2)

- **Blocked Call**: Call which cannot be completed at time of request, due to congestion.
  - Also referred to as a **lost call**.
- **Grade of Service (GOS)**: A measure of congestion which is specified as the probability of a call being blocked (for Erlang B).
  - The AMPS cellular system is designed for a GOS of 2% blocking.
    - This implies that the channel allocations for cell sites are designed so that, on average, 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour.

Erlang B Formula

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

Call blocking probability

m = Number of trunked channels

$A =$ traffic intensity or load [Erlangs]

$$= \frac{\lambda}{\mu}$$

$\lambda =$ Average # call attempts/requests per unit time

$\frac{1}{\mu} = H =$ Average call length

In MATLAB, use
```
erlangb(m,A)
```

We use the MATLAB code from http://infohost.nmt.edu/~borchers/erlang.html to evaluate the Erlang B formula.

Exercise

- Use the Erlang B formula to find P_b for the values of m and A specified in the table below.

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

m \ A	1	2	∞
1	1/2	2/3	1
2	1/5		

$$P_b = \frac{\frac{A^1}{1!}}{\frac{A^0}{0!} + \frac{A^1}{1!}} = \frac{A}{1+A}$$

$$P_b = \frac{\frac{A^2}{2!}}{\frac{A^0}{0!} + \frac{A^1}{1!} + \frac{A^2}{2!}} = \frac{\frac{A^2}{2}}{1 + A + \frac{A^2}{2}} = \frac{A^2}{2 + 2A + A^2}$$

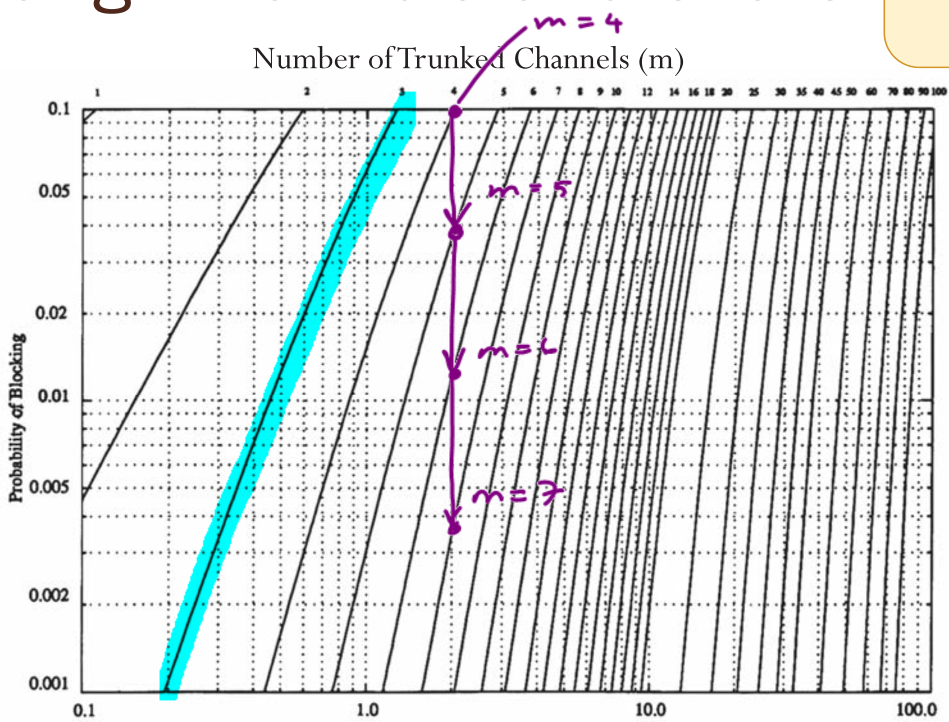
M/M/m/m Assumption

- **Blocked calls cleared**
  - No queuing for call requests.
  - For every user who requests service, there is no setup time and the user is given immediate access to a channel if one is available.
  - If no channels are available, the requesting user is blocked without access and is free to try again later.
- **Calls arrive as determined by a *Poisson process*.**
- There are memoryless arrivals of requests, implying that all users, including blocked users, may request a channel at any time.
- There are an **infinite** number of users (with finite overall request rate).
  - The finite user results always predict a smaller likelihood of blocking. So, assuming infinite number of users provides a conservative estimate.
- **The duration of the time that a user occupies a channel is *exponentially distributed***, so that longer calls are less likely to occur.
- There are m channels available in the trunking pool.
  - For us, m = the number of channels for a cell (C) or for a sector

Erlang B Formula and Chart

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

Number of Trunked Channels (m)



Traffic Intensity in Erlangs (A)

(log-log plot)

Need $P_b \leq 0.005$

Example 1

- How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?
  - $m =$ (a) 5 $\xRightarrow{\text{Erlang B}}$ $A = 1.13$ [Erlangs] $\Rightarrow n = \left\lfloor \frac{1.13}{A_u} \right\rfloor = 11$ users
  - $m =$ (b) 10 $\xRightarrow{\text{Erlang B}}$ $A = 3.96$ [Erlangs] $\Rightarrow n = \left\lfloor \frac{3.96}{A_u} \right\rfloor = 39$ users
- Assume each user generates $A_u = 0.1$ Erlangs of traffic.

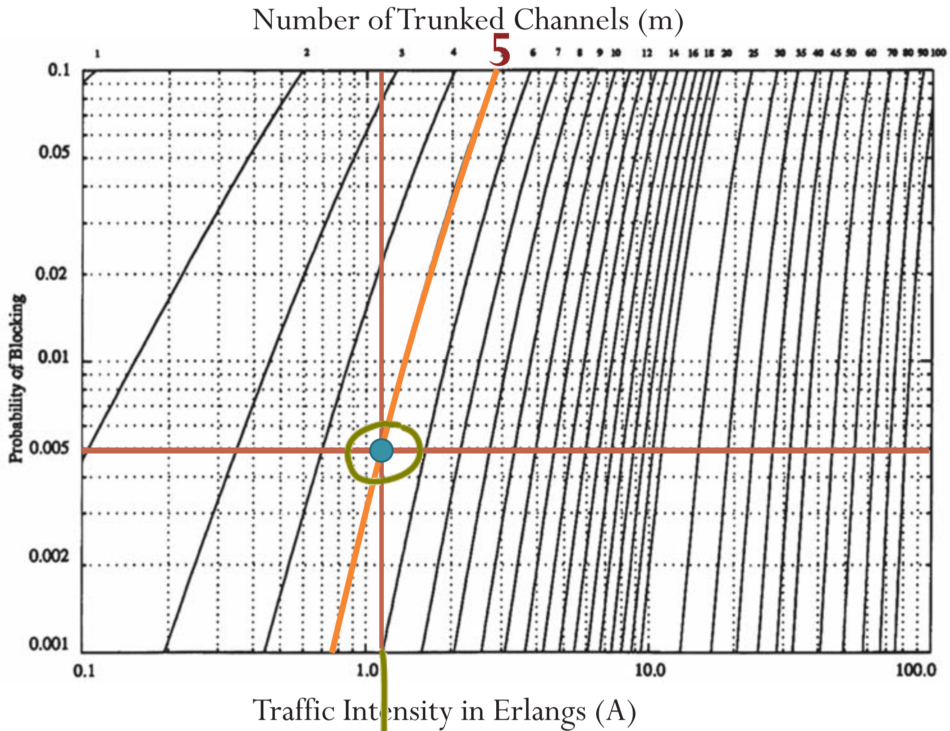
For example,

$\lambda_u = 6$ times/day

$H = \frac{1}{\mu} = 24$ min on average for each call

$$\left.\begin{array}{l}\lambda_u = 6 \text{ times/day} \\ H = \frac{1}{\mu} = 24 \text{ min}\end{array}\right\} A_u = \lambda_u \times H = \frac{6}{\cancel{24} \times 60} \times \cancel{24} = 0.1$$

Example 1a



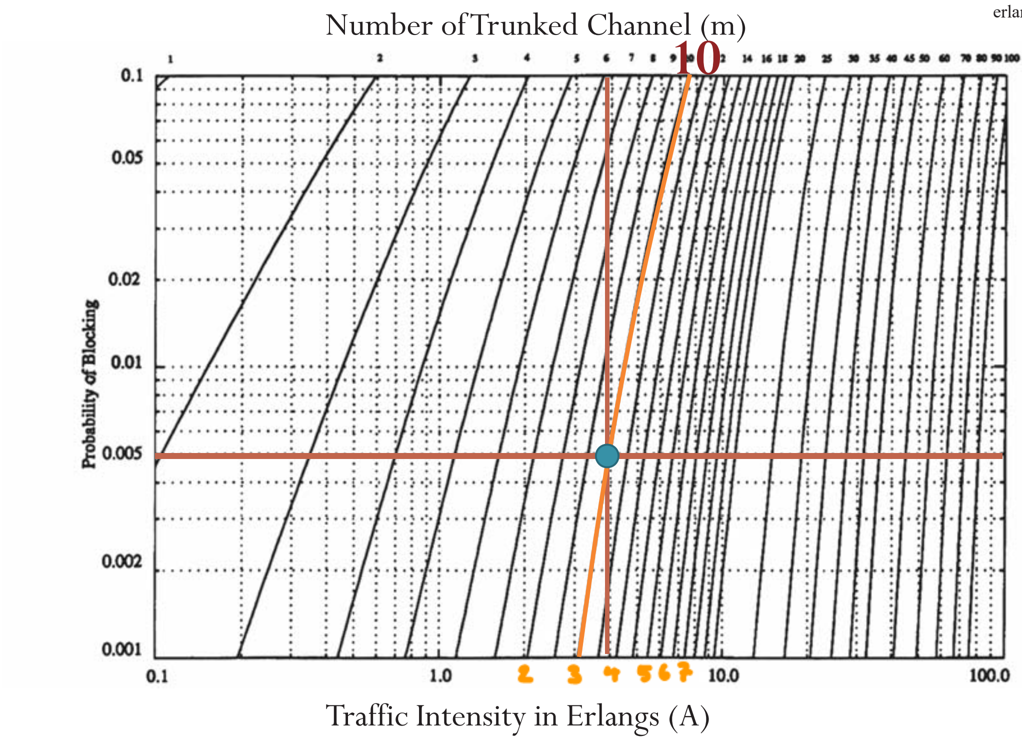
$\rightarrow A \approx 1.13$

~~$A \approx 1 \Rightarrow n \approx 10$ users~~

Example 1b

$\mathrm{erlangb}(10,3.5) = 0.0023$
$\mathrm{erlangb}(10,3.8) = 0.0039$
$\mathrm{erlangb}(10,3.9) = 0.0046$
$\mathrm{erlangb}(10,3.95) = 0.0049$
$\mathrm{erlangb}(10,3.96) = 0.0050$
$\mathrm{erlangb}(10,3.97) = 0.0051$
$\mathrm{erlangb}(10,4) = 0.0053$

A



$A \approx 4 \Rightarrow n \approx 40$ users

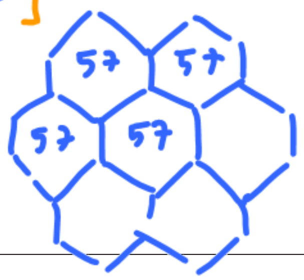
Example 2.1

- Consider a cellular system in which
  - an average call lasts two minutes $\rightarrow H = \frac{1}{\mu} = 2$ mins
  - the probability of blocking is to be no more than 1%. $P_b \leq 0.01$
- If there are a total of 399 traffic channels for a seven-cell reuse system, there will be 57 traffic channels per cell. S; $N = 7$
- From the Erlang B formula, can handle 44.2 Erlangs or **1326 calls per hour**.

$$m = \left\lfloor \frac{S}{N} \right\rfloor = \left\lfloor \frac{399}{7} \right\rfloor = 57$$

$$A = \lambda H$$

$$\lambda = \frac{A}{H} = \frac{44.2}{2 \text{ min}} = 22.1 \frac{\text{calls}}{\text{min}} = 1326 \frac{\text{calls}}{\text{hour}}$$

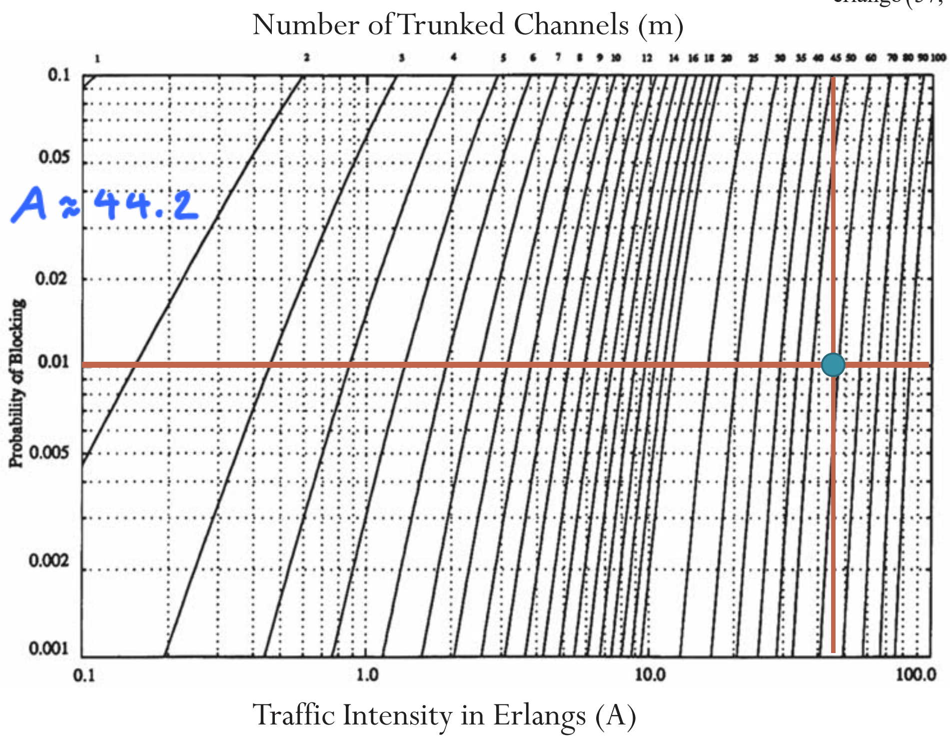


[Rappaport, 2002, Ex 3.9, p 92]

$$\mathrm{erlangb}(57, 44) = 0.0094$$
$$\mathrm{erlangb}(57, 44.2) = 0.0099$$
$$\mathrm{erlangb}(57, 44.3) = 0.0102$$
$$\mathrm{erlangb}(57, 44.5) = 0.0109$$
$$\mathrm{erlangb}(57, 45) = 0.0125$$

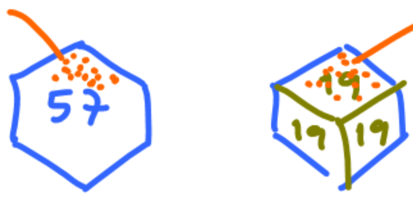
Example 2.1: Erlang B

$m = 57$
$P_b = 0.01$ $\Big\} \Rightarrow A \approx 44.2$



20 users here are OK

not so



Example 2.2

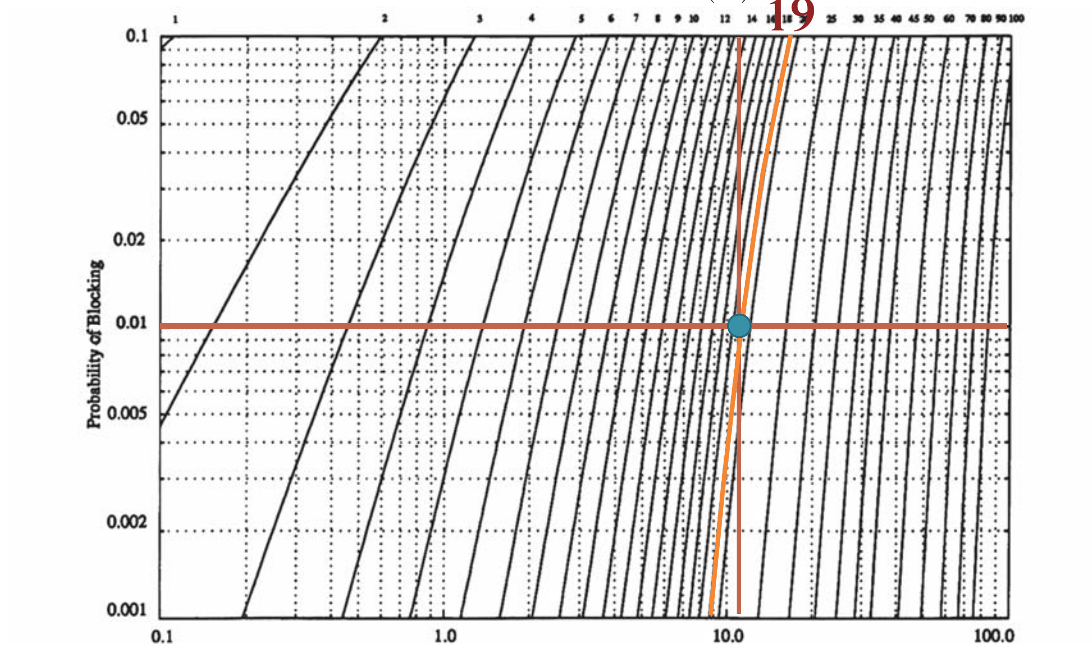
- Now employing **120° sectoring**, there are only m = $\lfloor 57/3 \rfloor$ = 19 channels per sector.
- For the same probability of blocking and average call length, each sector can handle 11.2 Erlangs or 336 calls per hour.
  $11.2 \times 3 = 33.6$ Erlangs (per cell) vs. 44.2 when no sectoring
- Since each cell consists of three sectors, this provides a cell capacity of 3 × 336 = **1008 calls per hour**, which amounts to a 24% decrease when compared to the unsectored case.
- Thus, sectoring decreases the **trunking efficiency** while improving the SIR for each user in the system.

[Rappaport, 2002, Ex 3.9, p 92]

Example 2.2: Erlang B

erlangb(19,11) = 0.0085
erlangb(19,11.2) = 0.0098
erlangb(19,11.3) = 0.0105
erlangb(19,11.5) = 0.0120
erlangb(19,12) = 0.0165

Number of Trunked Channel (m)



Traffic Intensity in Erlangs (A)

Erlang B Trunking Efficiency

Number of Channels m	Capacity (Erlangs) for GOS = 0.01 (1%)	= 0.005	P_b = 0.002	= 0.001 (0.1%)
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

×2   × >2   A

[Rappaport, 2002, Table 3.4]

Summary of Chapter 2: Big Picture

S = total # available duplex radio channels for the system

Frequency reuse with **cluster size N**

"Capacity" $C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}$

Tradeoff

Path loss exponent

$$\frac{S}{I} \approx \frac{kR^{-\gamma}}{K \times \left(kD^{-\gamma}\right)} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma} = \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

m = # channels allocated to each cell.

Omni-directional: K = 6
120° Sectoring: K = 2
60° Sectoring: K = 1

Trunking

m = # trunked channels

Call blocking probability $P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}$.

Erlang-B formula

λ = Average # call attempts/requests per unit time

$A =$ traffic intensity or load [Erlangs] $= \frac{\lambda}{\mu}$

$\frac{1}{\mu} = H$ = Average call length

Example 3: System Design (1)

- 20 MHz of total spectrum.
- Each simplex channel has 25 kHz RF bandwidth.
- The number of duplex channels:

$$S = \frac{20 \times 10^6}{2 \times 25 \times 10^3} = 400 \text{ channels}$$

- $\gamma = 4$
- Design requirements:
  - SIR ≥ 15 dB
  - $P_b \leq 5\%$
- Goal: Maximize the number of users that can be supported by the system.
- Question:
  - N = ?
  - Should we use sectoring?

Two-step plan
① Use SIR to determine N
② Use P_b to determine A
→ compare different options.

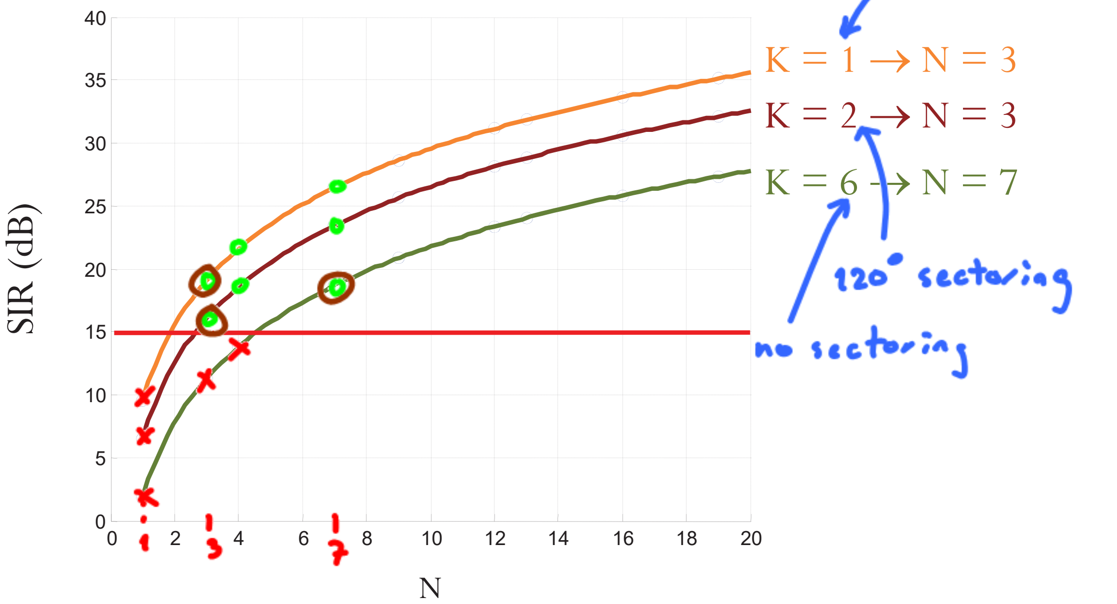
```
clear all; close all;
y = 4;
figure; grid on; hold on;
for K = [1,2,6]
    N = [1, 3, 4, 7, 9, 12, 13, 16, 19]; 21
    SIR = 10*log10(1/K*((sqrt(3*N)).^y))
    plot(N,SIR,'o')
    N = linspace(1,20,100);
    SIR = 10*log10(1/K*((sqrt(3*N)).^y));
    plot(N,SIR)
end
```

Example 3 (2)

- SIR ≥ 15 dB

$$SIR \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

4



Make sure that you understand where numbers in this table come from!

Example 3 (3)

	Omnidirectional	120° Sectoring	60° Sectoring
K	6	2	1
N	7	3	3
SIR [dB]	18.7	16.1	19.1
#channels/cell $= \lfloor \frac{S}{N} \rfloor$	$\lfloor 400/7 \rfloor = 57$	$\lfloor 400/3 \rfloor = 133$	$\lfloor 400/3 \rfloor = 133$
#sectors/cell	1	3	6
m = #channels/sector	57	$\lfloor \frac{400}{3}/3 \rfloor = 44$	$\lfloor \frac{400}{3}/6 \rfloor = 22$
A [Erlangs]/sector	51.55	38.56	17.13
A [Erlangs]/cell	51.55	38.56×3 = 115.68	17.13×6 = 102.78
#users/cell	18558	41645	37001

Erlang B $P_b = 0.05$

max

Assume that each user makes 2 calls/day and 2 min/call on average → 1/360 Erlangs.

Conclusion: With $\gamma = 4$, SIR ≥ 15 dB, and Pb ≤ 5%,
120° sectoring with cluster size N = 3 should be used.

Example 3 (4): Remarks

	Omnidirectional	120° Sectoring	60° Sectoring
K	6	2	1
N	7	7	7
SIR [dB]	18.7	23.43	26.44
#channels/cell	$\lfloor 400/7 \rfloor = 57$	$\lfloor 400/7 \rfloor = 57$	$\lfloor 400/7 \rfloor = 57$
#sectors/cell	1	3	6
m = #channels/sector	57	$\left\lfloor \frac{400}{7}/3 \right\rfloor = 19$	$\left\lfloor \frac{400}{7}/6 \right\rfloor = 9$
A [Erlangs]/sector	51.55	14.31	5.37
A [Erlangs]/cell	51.55	$14.31\times 3 = 42.94$	$5.37\times 6 = 32.22$

For the same N, we see that 120° sectoring and 60° sectoring give much better SIR. However, sectoring reduces the trunking efficiency and therefore suffer reduced value of A.

	Omnidirectional	120° Sectoring	60° Sectoring
K	6	2	1
N	7	7	7
SIR [dB]	18.7	23.43	26.44
#channels/cell	$\lfloor 400/7 \rfloor = 57$	$\lfloor 400/7 \rfloor = 57$	$\lfloor 400/7 \rfloor = 57$
#sectors/cell	1	3	6
m = #channels/sector	57	$\left\lfloor \frac{400}{7}/3 \right\rfloor = 19$	$\left\lfloor \frac{400}{7}/6 \right\rfloor = 9$
A [Erlangs]/sector	51.55	14.31	5.37
A [Erlangs]/cell	51.55	$14.31\times 3 = 42.94$	$5.37\times 6 = 32.22$

Idea: The values of SIR are too high for the cases of 120° sectoring and 60° sectoring. We can further reduce the cluster size. This increases the number of channels per cell and hence per sector.

	Omnidirectional	120° Sectoring	60° Sectoring
K	6	2	1
N	7	3	3
SIR [dB]	18.7	16.1	19.1
#channels/cell	$\lfloor 400/7 \rfloor = 57$	$\lfloor 400/3 \rfloor = 133$	$\lfloor 400/3 \rfloor = 133$
#sectors/cell	1	3	6
m = #channels/sector	57	$\left\lfloor \frac{400}{3}/3 \right\rfloor = 44$	$\left\lfloor \frac{400}{3}/6 \right\rfloor = 22$
A [Erlangs]/sector	51.55	38.56	17.13
A [Erlangs]/cell	51.55	$38.56\times 3 = 115.68$	$17.13\times 6 = 102.78$